NOT TO BE FURTHER COPIED OR REPRODUCED

(* BATCHES *)

7z

Means and Midmeans With Less Arithmetic

[almost optional]

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7z

Means and Midmeans With Less Arithmetic

[almost optional]

Throughout this account we have fled arithmetic where we could. In dealing with batches we have been fairly successful. The trimean is easy -- because stem-and-leaf is there to help us. We copy, and rearrange, and count, but the only arithmetic is two halvings. What could be better?

There are people who use means routinely. We have admitted that there are occasional places where we will need to use them ourselves. Making their use easier is worth some attention.

An adding machine of any sort can help a lot. Even a modern computer can help if there is access to it through an interactive terminal and a sufficiently simple system. But these are not always available.

We are going to have -- at least occasionally, perhaps frequently -- to calculate means by hand. Anything that eases the pain is good. Hence this chapter.

Midmeans, of course, are just means of smaller selected batches -- means of batches that are middle parts of other batches.

7zA. Finding means easily

Arranging numbers in order is easy for those who use stem-and-leaf techniques, even if it can be much more work for those who do not. Finding the median of 100 -- or 218 -- values need not, as we have seen, be a great effort.

(* adding up *)

Finding means tends to be more work, but it too can be made easier. Let us look at the 10 bowling scores and see how to make finding their mean easier. Our main attention needs to be given to adding them up. Exhibit **1** shows a simple but widely useful technique. Panel A separates the digits and adds each column of digits separately. Panel B puts the answers together. Panel C shows how we expect to write the whole computation down. In it, too, we added up only one column at a time.

In adding single columns, it can pay, if your mental adder works smoothly enough, add in two figures at a time. The right-hand column of exhibit 1, for instance, would then be done as follows:

tacit	**audible**
(3+4 = 7)	7
(5+8 = 13)	7 and 13 make 20
(3+7 = 10)	20 and 10 make 30
(1+4 = 5)	30 and 5 make 35
(2 = 2)	35 and 2 make 37

with further condensation to: "7 and 13 is 20, and 10 is 30, and 5 is 35, and 2 is 37".

(* the stem-and-leaf case *)

The principle of adding separate digits applies neatly to stem-and-leaf presentations, as shown in exhibit **2**. This display also gives two levels of short-cutting:

exhibit 1 of chapter 7z

How to add easily

A) The INDIVIDUAL COLUMNS ADDED SEPARATELY

Data	= Hundreds	+ Tens	+ Units
193	1	9	3
214	2	1	4
185	1	8	5
198	1	9	8
203	2	0	3
207	2	0	7
191	1	9	1
181	1	8	1
210	2	1	0
192	1	9	2
(sums)	14	54	34

B) The ACCUMULATION

14 hundreds + 54 tens + 34 = 1977

C) The CONDENSED FORM

```
 193
 214
 185
 198
 203
 207        (alternate)
 191         (version)
 184
 210
 192
 ---        ----
  37        1437
 54          54
14          ----
----        1977
1977
```

D) NOTE

Delaying the carry -- this is called "bookkeeper's addition" -- makes checking easier, and, surprisingly, saves a noticeable amount of mental effort.

exhibit 2 of chapter 7z

Use of separate digit addition in a stem-and-leaf

A) The BOWLING SCORES again

21	40
20	37
19	3812
18	54

B) SPLIT SUMS

4 + 0 + 3 + 7 + 3 + 8 + 1 + 2 + 5 + 4 = 37

21 twice + 20 twice + 19 4 times + 18 twice = 194

C) COMBINATION

194 tens + 37 units = 1977

D) END of FIRST VERSION

$$\text{mean} = \frac{1977}{10} = 197.7$$

E) TAKE the STEMS as "excess over 18"

3 twice + 2 twice + 1 4 times

sum = 14

F) COMBINING "excess over 18"

14 tens and 37 = 177

mean (of excess) = 17.7

G) FINDING MEAN

18 tens + 17.7 = 197.7

(base + mean excess)

- one in which we add up the full stems.

- one in which we work out the mean excess over some convenient base value, here taken as 18 tens (= 180).

We expect to use such a device whenever we have more than a few numbers to add up and these numbers appear in a stem-and-leaf pattern. Exhibit 3 shows a larger example, with positive and negative excesses.

If we use these techniques, adding bothers us much less.

(* easier division *)

When we want the mean of a medium-sized or large batch, we may have a reason to make division easier. Exhibit 4 shows one way to do this. We invest in calculating one to nine times the divisor (and 10 times it as a check) -- which we can do by successive addition -- once for all, so that we can merely subtract the correct multiple at each stage of division.

These aids to addition and division may seem extremely simple, but if you try finding the mean of, say, 43 values without and with them, you are likely to appreciate their value.

7zB. Finding midmeans easily

The first step in finding a midmean is to put the values in increasing -- or decreasing -- order. Once we have more than a few values, the easiest way to get them in order is to make a stem-and-leaf.

exhibit 3 of chapter 7z

The mean of the logs of the 1960 populations of the 50 states

A) STEM-and-LEAF and SUBTOTALS

Stem-and-leaf		Subtotals for leaves	Calculation for stems: Excess of stem over 20	Times ↓	Equals ↓
13	6	6	-7	1	-7
14	6	6	-6	1	-6
15	39	12	-5	2	-10
16	5	5	-4	1	-4
17	9	9	-3	1	-3
18	00333	9	-2	5	-10
19	3589	25	-1	4	-4
20					
21	155	11	1	3	3
22	445	13	2	3	6
23	4478	23	3	4	12
24	04589	26	4	5	20
25	1135	10	5	4	20
26	0004679	26	6	7	42
27	18	9	7	2	14
28	9	9	8	1	8
29	89	17	9	2	18
30	05	5	10	2	20
31					
32	02	2	12	2	24
		93			+13
	Sums	13			+13
		223		(50, check)	+143

Total excess $223(0.01) + 143(0.1) = 16.63$ = sum of 50

B) FINISH of CALCULATION

$$\text{mean excess over } 2.0 = \frac{16.63}{50} = .3326, \text{ say } .33$$

$$\text{mean} = 2.33 = 2.0 + .33$$

exhibit 4 of chapter 7z

Investing in calculating multiples to make division faster

A) The TASK

Find $\frac{12468975}{113}$

B) INVESTMENT MULTIPLES -- by successive addition

1	113
2	226
3	339
4	452
5	565
6	678
7	791
8	904
9	1017
(10)	(1130, check)

C) The SUBTRACTION

```
12468975
113
 116
 113
   389
   339
    507
    452
     555
     565
      -
```

answer: 110345

D) FINDING MULTIPLES EASILY -- alternate method

Pattern: 1+1 = 2, 2+2 = 4, 4+4 = 8, 4+2 = 6, 2+1 = 3, 4+1 = 5, 6+1 = 7, 8+1 = 9

In example: 2(113) = 226, 2(226) = 452, 2(452) = 904, 226+452 = 678, 226+113 = 339, 452+113 = 565, 678+113 = 891, 904+113 = 1017

This would have been worthwhile for the 10 bowling scores, where starting to find their midmean by scratching down

```
21|40
20|37
19|3812
18|54
```

would certainly not have lost time -- and would equally certainly have saved mental effort. Just where the break-even point comes depends on the individual. A good way to begin is to stem-and-leaf any batch of seven or more. You will soon learn whether you want to advance to six or retreat to eight. Larger batches -- if you have collected them yourself -- are likely to be already in stem-and-leaf form for convenience of handling.

For small batches, writing out the ordered values in detail may be helpful. But for larger ones it is easy to find hinges and midmean without leaving the stem-and-leaf format. Exhibit 5 reminds us how this goes. The first step is to find $\frac{1}{4}$ of the number in the batch, namely $6\frac{1}{4}$, which grows to 7, thus identifying the depth of the hinges as 7. The values of the hinges are then found to be 26 (larger than 24, 23, 18, 15, 13, and 5) and 51 (smaller than 52, 56, 57, 62, 63, and 86).

With the hinges settled, panel C repeats the part of the stem-and-leaf that reports middle values. The next steps are to find first the sum -- most easily separately for stems

exhibit 5 of chapter 7z

Numbers of days with snow-covered ground; 25 New York City winters beginning 1894-95 (Calculation of hinges and easy midmean from stem-and-leaf)

A) STEM-and-LEAF

	to 1	8	6
		7	
	to 2	6	23
	to 7 H1	5	1267
	to 10	4	489
25 ✓	to 17	3	1446679
	to 8 H6	2	3469
	to 4	1	358
	to 1	0	5

B) LOCATION of HINGES

25/4 = 6.25 grows to 7, so "7th in" are hinges.
Middle count will be 25-7-7 = 11.

Hinges: 26 and 51

C) MIDDLE and MIDMEAN

(3)	4	489
(7)	3	1446679
(1)	2	9
(11, check)		

sum of leaves = 67 = 4+8+9+1+4+...+9+9
sum of stems = 35 = 3(4) + 7(3) + 1(2)
midsum = 417 = 67 + 10(35)
midmean = 38 = 417/11

(Cont'd)

exhibit 5 (Cont'd)

D) CONDENSED VERSION

to 1	8	6
	7	
to 3	6	23
to 7	5	1267
to 10	4	
to 17	3	1446679
to 8	2	3469
to 4	1	358
to 1	0	5
	25	
	H	26
	H	51
	mid	38

25 v

(middle)
sum of stems = 35
sum of leaves = 67

4	489
3	1446679
2	9

417 = 35 tens + 67
38 = 417/11 = midmean

E) SOURCE

The World Almanac and Encyclopedia for 1921, page 518.

and leaves -- and then the mean of these middle values, which is the midmean.

Exhibit 6 gives another example -- one that illustrates another point:

- since the stems are all large, 51 or 52, it is easier arithmetic to calculate the mean excess of the middle values over 51, and then add 51 to the result in order to find the mean of the middle values, which is the midmean.

exhibit 6 of chapter 7z

Annual mean temperatures in New York City for
48 calendar years beginning in 1872

A) STEM- and-LEAF WITH CONDENSED CALCULATION -- unit 0.1°F

				(middle)
	to 2	54	33	
	to 8	53	555788	
	to 9	53	1	
	to 18	52	566667999	52 | 577778
48v	to 23	52	02223	52 | 02223
	to 25	51	55566899	51 | 55566899
	to 17	51	022333344	51 | 33344
	to 8	50	566	
	to 5	50	2	(240, check)
	to 4	49	889	
		49		
	to 1	48	6	
		48		
		12.5H	51.3	
		12.5H	52.8	

B) ARITHMETIC DETAIL

Middle count = 6+5+8+5 = 24 (check)

Middle leaves: sum = 115(0.1) = 11.5

Excess of middle stems over 51: sum = (6+5)1 = 11

Excess of middle values over 51: sum = 11+11.5 = 22.5

mean = 0.94 = 22.5/24

Midmean = 51.94 = 51+0.94

C) SOURCE

The World Almanac and Encyclopedia for 1921, page 578.

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7z H. How far have we come?

The only way to gain on hand addition is to do it digit by digit with delayed carry. This makes it easier to check and freer from strain. Some will find picking up each digit from two numbers at a time helpful.

Stem-and-leaf displays invite calculating separating with separate digits. They also encourage using differences from a chosen origin.

Division can sometimes be eased by calculating all 10 multiples of the divisor -- or a skeleton pattern of them -- before beginning the division.

The general maxim is simple;

- if you are going to need it, even as an intermediate step, it is not a waste of time to write it down at the time this is easiest to do.

Midmeans are a special case, where the stem-and-leaf is already an essential tool in the first step of identifying the values to be added.